MA 234, Spring 2016

Assignment 4

This homework is due Friday Feb 19.

There are total 45 points in this assignment. 40 points is considered 100%. If you go over 40 points, you will get over 100% for this homework (but not over 115%) and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper *and give credit to your collaborators in your pledge*. Your solutions should exhibit your work and contain full proofs. Bare answers will not earn you much.

This assignment covers Sections 2.1–2.3 of Textbook.

Careful! This homework may take longer than the previous ones. Plan accordingly.

(1) [7pt] Find the images of the mapping $w = (3+4i)z - 2 + i$ in each case, and sketch the mapping (that is, sketch the region and its image). In each case, identify three points of your choice and their corresponding images.
   (a) Find the image of the disk $|z-1| < 1$.
   (b) Find the image of the line $x = t, y = 1 - 2t$ for $-\infty < t < \infty$.
   (c) Find the image of the half-plane $\mathrm{Im}(z) > 1$.

(2) [10pt] Find the images of the mapping $w = z^2$ in each case, and sketch the mapping.
   (a) The horizontal line $\{(x,y) : y = 1\}$.
   (b) The vertical line $\{(x,y) : x = 2\}$.
   (c) The rectangle $\{(x,y) : 0 < x < 2,\ 0 < y < 1\}$.
   (d) The rectangle $\{(x,y) : -1 < x < 2,\ -2 < y < 1\}$.
   (e) The infinite strip $\{(x,y) : 1 < x < 2\}$.

(3) [10pt] Find and sketch the images of the following sets under the mapping $w = z^{\frac{1}{2}}$, the principal square root function.
   (a) $\{re^{i\theta} : r > 1,\ \frac{\pi}{3} < \theta < \pi\}$.
   (b) $\{re^{i\theta} : 1 < r < 9,\ 0 < \theta < \frac{2\pi}{3}\}$.
   (c) $\{re^{i\theta} : r < 4,\ -\pi < \theta < \frac{\pi}{2}\}$.
   (d) The horizontal line $\{(x,y) : y = 2\}$.
   (e) The vertical line $\{(x,y) : x = 4\}$ (*Hint:* $u^2 - v^2 = A$ (where $A \neq 0$ is an equation of a hyperbola with asymptotes $u = \pm v$).

(4) [5pt] Find the following limits using basic arithmetic properties of limits.
   (a) $\lim_{z\to 1} \frac{z^2+1}{z+1}$.
   (b) $\lim_{z\to 1} \frac{z^2-1}{z-1}$.
   (c) $\lim_{z\to i} \frac{z^4-1}{z-i}$.
   (d) $\lim_{z\to 1+i} \frac{z^2+z-2+i}{z^2-2z+1}$.
   (e) $\lim_{z\to 1+i} \frac{z^2+z-1-3i}{z^2-2z+2}$. (*Hint:* Factor[1] both quadratic polynomials as $(z-z_1)(z-z_2)$.)

— see next page —

[1]The following may help. (1) If z_1, z_2 are roots of $z^2 - bz + c$, then $z^2 - bz + c = (z-z_1)(z-z_2)$. (2) If you know one root, you can find the other by observing that $z_1 + z_2 = b$.

(5) [5pt] Let $f(z) = \frac{xy^3}{x^2+y^6} + i\frac{x^3y}{x^6+y^2}$.
   (a) Find $\lim_{z\to 0} f(z)$ as z approaches 0 along the vertical straight line $x = 0$.
   (b) For every real k, find $\lim_{z\to 0} f(z)$ as z approaches 0 along the straight line $y = kx$.
   REMARK. Note that (a) and (b) together cover all possible straight lines on a complex plane passing through 0.
   (c) Find $\lim_{z\to 0} f(z)$ as z approaches 0 along any the cubic curve $x = y^3$.
   (d) What can you conclude about the limit of $f(z)$ as $z \to 0$?

(6) [3pt] Does $\lim_{z\to -4} \mathrm{Arg}(z)$ exist? Why? (*Hint:* Use polar coordinates and approach -4 from the upper and lower half-planes.)

(7) [5pt] (This problem is something I forgot to include in HW3. It's about material of Section 1.5 of Textbook.) Find all roots
   (a) of degree 4 of -1.
   (b) of degree 6 of $8i$.
   (c) of degree 7 of e^{11i}.